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ABSTRACT

A diagnostic computational test given to 22 arithmetic teachers in 1930 was administered to a similar group of 35 elementary school teachers in 1973. The purpose was to compare and contrast computational errors made by the teachers. Findings revealed that 1973 teachers possess, on the average, more computational skills than the 1930 teachers. Both populations had trouble with fractions, decimals, and percentage problems. Discussions are presented as to the legitimacy of comparing the two populations and to the implications of the results. (Author/LS)

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An Analysis of Computational Errors Made By Teachers of Arithmetic: 1930, 1973

by

Theodore A. Eisenberg

In 1932 Guiler reported an analysis of computational errors made by teachers of arithmetic. This analysis was made by categorizing the teacher's responses to questions on a diagnostic computational test.[1] The twenty-two teachers in the study were then enrolled in Professor Guiler's course in educational and mental tests during the summer of 1930 at Miami University, Oxford, Ohio. The exam used was the Guiler-Christofferson Diagnostic Survey Test in Computational Arithmetic and covered five areas of computation: 1) whole numbers, 2) fractions, 3) decimals, 4) practical measurements, and 5) percentage. Each area consisted of ten problems worth one point each if correctly solved. All teachers in the study had taught arithmetic, along with other subjects, in the public schools of Ohio during the 1930 school year.

Education in general, but specifically mathematics education, have undergone drastic changes since 1930. Hopefully, things changed for the better. Requirements to graduate from college and for certification to teach in this decade are much more stringent than they were in the 1930's. In general, current teachers are much more sophisticated than their 1930 counterpart. In order to be certified to teach in an elementary school in the state of Ohio in 1930, one needed only to have graduated from a "first grade" high school (or equivalent) and from a two year normal school curriculum.[2] Guiler reported : "All these in service teachers were high school graduates and had had one or

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more years of professional training." However, in order to be certified to teach in a public elementary school in Ohio in 1973, one needed a bachelors degree from a four year accredited institution along with specific academic and professional courses.[3]

In many ways the 1930's can be considered infancy days in the mathematics education discipline. Perusal of the journals during that period reveal that educators then, as now, worried about meaningful instruction and a development in the populous of a "feeling" for mathematics. But the 1930 era predates the specific subject matter emphases and the tremendous school mathematics curricula reforms similar to the ones witnessed in the "modern math" movement. With the aide of hindsight, we realize that education during the past forty years has been vicissitudinous. We have followed many false prophets and messiahs, only to realize later that we were misguided. We have adopted educational theories and practices but these too have not sustained the test of time. Nevertheless, it is believed that inroads have been made into meaningful mathematics instruction.

A time honored goal has transcended the near half century period since the Guiler study. A common theme of all programs, past and present, is that elementary school teachers should possess the ability to compute. Indeed, possessing the ability to compute is the unavoidable requisite for understanding the deeper underlying concepts of the real number system, which is one of the main themes of the modern math movement. It was the purpose of this study to compare and contrast computational errors made by teachers of arithmetic in 1930 and 1973.

Procedures

Twenty-five of the fifty items on the Guiler-Christofferson diagnostic test were given to thirty-five elementary school teachers all of whom were enrolled in a graduate level mathematics methods course in the School of Education at Ohio State University, Columbus, Ohio. Only those students who were then teaching arithmetic in elementary grades were elegible to participate in the study. The class at OSU had at least as many persons teaching at each grade level as did the population in the Guiler study. If there were more teachers at a particular grade level than required by the Guiler study, the appropriate number were randomly selected. Thus, the grade level distributions for the 1930 and 1973 populations were identical. There was one ninth grade teacher; four in the eighth grade; two in the sixth grade; one in the fifth grade; four in the fourth grade; three in the third grade; five in the second grade; and two in the first grade for each population of teachers. The two populations were also similar in that they both represented teachers who were highly motivated: the teachers were enrolled in in-service courses. It should be mentioned, however, that it was literally impossible to have precisely identical teacher populations. Elementary school teachers who had only high school diplomas did exist in 1973. Cognizant of that constraint, the two populations were identical relative to their time in history.

Guiler reported 25 of the 50 questions on his diagnostic test and an item by item response for each of the 22 teachers in his study. Because of this detailed reporting of the data, comparisons of the 1930, 1973 populations were possible on the 25 reported questions. For those questions the response patterns of the 1973 population of teachers were

compared to those for the 1930 population. Comparisons of mean scores and the types of errors made in computation were analyzed for each area on the exam.

Findings

The test questions and the number and percentage of teachers incorrectly answering each question on the exam are recorded for the respective populations in Table 1.

Insert Table 1 About Here

A study of this table indicates that in general 1973 teachers can compute more accurately than their 1930 counterpart. But are the observed differences significant? In an effort to answer this question, we consider an item by item response for each population. This is listed in Table 2. From this we construct frequency distributions of the items missed by each population on the entire test and the specific subtests. It is also possible to compare the response patterns of the top 27%, the bottom 27%, as well as the middle groups of each population. Using one-tailed t-tests we can statistically compare the performance of the two populations relative to specific sections on the exam.

Insert Table 2 About Here

Overall, the 1973 teachers did significantly better on the exam than the 1930 teachers. However, the performance of the top 27% of the 1930 population did not significantly differ from that of the top 27% of the 1973 population; similar results were found for the bottom 27% of the populations. The main differences occured within the middle

groups. The middle group of 1973 teachers was significantly more accurate in computation than their 1930 counterpart, and this difference was so drastic that it was the cause of the observed significant difference in the performance of the two entire populations. Table 3 summarizes the findings. This implies that the "average" 1973 teacher was significantly more accurate in computational skills than his 1930 counterpart. This fact will be elaborated upon in the ensuing discussion.

Insert Table 3 About Here

A more interesting picture develops when focusing on the specific categories of the exam. Looking at response patterns for the entire population, significant differences were observed in only two areas: whole numbers and measurements. No meaningful differences in performance were encountered in the other areas on the exam: fractions, decimals and percentage. In fact, 1973 teachers tended to do worse on percentage problems than 1930 teachers, but the observed difference was not statistically meaningful. The representative t-tests are listed in Table 4.

Insert Table 4 About Here

When comparing the performance of the top group of 1930 teachers with the top group of 1973 teachers, and the respective bottom groups of each population, one notices striking similarities. For the top groups, significant differences in performance occured only with whole numbers and percentage-type problems. For the bottom groups, whole numbers and measurement problems were the only categories in which the

populations meaningfully differed. In each case, 1973 teachers displayed more skill in working the representative problems of the categories than the 1930 teachers. Interestingly, the middle group of teachers significantly differed on only the fractions and measurements categories of the test. These findings are displayed in Table 5.

Insert Table 5 About Here

It should also be noted that for the 1930 population, the top group of teachers displayed significantly more skill in computation than the bottom group of 1930 teachers on each category of the exam. This was also true for the 1973 population except for the category on measurement. On this category of questions, teachers in the bottom group performed as well as those in the top group. Also, no pattern was observed on computational skill and grade level taught for the 1973 population. Second grade teachers tended to do as well as, and as poorly as eighth grade teachers on each category of the exam. (Guiler did not report a matching of teachers with grade level, consequently, such a comparison could not be made for his population.)

Discussion

A number of salient facts can be gleaned from Table 2. Teachers still have a tremendous amount of trouble with decimals and percentage-type problems. Yet, progress has been made particularly in the areas of whole numbers and fractions and most notably with measurement. But there is still room for improvement. We should be comparing the performance of our teachers to absolute scales, not to relative ones.

It is true that teachers in 1973 were more accurate computers than teachers in 1930--but it is frightening to think that at least 59%of current day teachers can not answer the question:

"An increase from $16 to $20 is _____% increase."

Moreover, when one thinks that these teachers will expose children to the underlying meaning of the concept of fractions, decimals, and percentage, the present day state of computational skills of our teachers becomes even more alarming.

Mention was made that it is really not legitimate to compare the two populations because a present day counterpart to the 1930 teacher does not exist. Guiler compared the responses of the teachers to 1930 college student norms. The teachers did significantly better than 1930 college freshmen and almost as well as college sophomores. But in 1973, all teachers were college graduates, and some even had masters degrees. Against that backstop, the meaningful differences observed become even more suspicious and the worth of the "mathematical revolution" in educating elementary school teachers with respect to computational skills suddenly becomes dubious.

Studies such as this seem necessary in order to assess the long range emphases of our educational programs. However, there are definite constraints to such comparitive studies. Looked at pessimistically, the findings are in accord with similar studies in this area,[4],[5] and we are led to conclude that the present generation of teachers are only slightly more sophisticated in mathematical skills than those of past generations.

Summary

This study showed that elementary school teachers in 1973 possess on the average more computational skills than elementary school teachers of 1930. It also revealed that both populations had trouble with fractions, decimals, and percentage-type problems. The desire to have teachers possess computational skills has remained constant throughout mathematics education history. The Guiler-Christofferson test was representative of these desired skills. (Question #19 was an exception to this -- pecks and bushels are no longer common units of dry measure.) We can test the mathematical state of our teachers by looking at problems similar to these -- and one fact is as apparent now as it was a half century ago: there is much, much room for improvement.

References

[1] Guiler, W.S. Computational Errors Made by Teachers of Arithmetic. The Elementary School Journal, September, 1932, 51-58.

[2] Laws Relating to Teachers Certificates. Ohio State Department of Education, 1930.

[3] Laws and Regulations Concerning Teacher Education and Certification. Ohio State Department of Education, 1973.

[4] Glennon, V.J. A Study of the Growth and Mastery of Certain Basic Mathematical Understandings on Seven Educational Levels. Harvard Educational Review, Winter, 1945, 69 (3), 62-64.

[5] Leonard, H.A. Difficulties Encountered by Elementary Algebra Students in Solving Equations in One Unknown -- A Diagnosis of Errors and a Comparison After Forty Years. Unpublished Doctoral Dissertation, Ohio State University, Columbus, Ohio, 1966.

Table 1

Test Questions and Number and Percentage of Teachers
From the 1930 and 1973 Populations Incorrectly Answering Test Items.

Test Items	Teachers Missing Items: 1930 Population Number	1930 Population Percent	1973 Population Number	1973 Population Percent
Part I - Whole Numbers:				
1. Add: 2689, 7655, 7974, 3279, 8868, 2697...	7	32	6	27
2. Subtract 95849 from 185744	8	36	2	9
3. Multiply 4608 by 89	8	36	4	18
4. Divide 31625 by 4 and show remainder	5	23	3	14
5. Divide 67092 by 86 and show remainder	13	59	4	18
Part II - Fractions:				
6. Add: 7 2/3, 14 1/8, 23 1/2	8	36	3	14
7. Subtract 6 8/9 from 12 1/6	15	68	3	14
8. Multiply 3 3/5 by 1 1/3	4	18	6	27
9. Divide 7 1/2 by 2 1/3	7	32	8	36
10. Change 8 4/9 to an improper fraction	1	5	2	9
Part III - Decimals:				
11. Copy and add: 89.8, 268.75, 76, and 35.6.	4	18	6	27
12. Copy and subtract: 3.83 from 8.6	3	14	2	
13. Place a decimal point in the following product: 7.06 x 2.7 = 19062	0	0	0	0
14. Place a decimal point in the following quotient: 986.79 ÷ 2.667 = 37	11	50	11	50
15. Change 11/12 to a decimal. Carry answer to three decimal places	7	32	3	14
Part IV - Practical Measurements:				
16. Copy and add (give answer in lb and oz.): 2 lb.; 3 lb. 8 oz.; 10 oz.; 2 lb. 4 oz. ..	6	27	1	5
17. Copy and subtract (give answer in hr. and min.): 2hr. 42 min. from 8 hr.	12	55	2	9
18. Copy and multiply (give answer in ft. and in.): 1 ft. 7 in. by 8	8	36	3	14
19. Copy and divide (give answer in bu. and pk.): 11 bu. 2 pk. by 4	13	59	18	82
20. Reduce 100 oz. to lb. and oz.	5	23	1	5
Part V - Percentage:				
21. 7 1/2% of $250 = $_____	5	23	5	23
22. 14 games is _____% of 16 games.....	6	27	7	32
23. 5 1/2% of _____ = 66	11	50	11	50
24. 20% less than $4.80 is $ _____	9	41	7	32
25. An increase from $16 to $20 is _____% increase	10	45	13	59

Table 2
Examples on Which Individual Teachers M

Teacher, 1930

Example	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	Total	1
Whole numbers:																								
1.....								X		X					X			X		X	X	X	7	
2.....							X	X				X		X		X			X	X		X	8	
3.....						X	X					X			X	X	X				X	X	8	
4.....								X				X			X		X					X	5	
5.....				X	X			X	X		X		X	X			X	X	X	X	X	X	13	
Number missed	0	0	0	1	1	1	2	4	1	1	1	3	1	2	3	2	3	2	2	3	3	5	41	0
Fractions:																								
6.....						X			X	X				X	X		X				X	X	8	
7.....								X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	15	
8.....				X			X								X							X	4	
9.....								X					X	X	X	X		X		X		X	8	
10.....																						X	1	
Number missed	0	0	0	1	0	1	1	2	2	2	1	1	2	3	4	2	2	2	1	2	2	5	36	0
Decimals·																								
11.....													X	X	X					X			4	
12.....													X				X			X			3	
13.....																							0	
14.....										X	X	X	X	X		X	X		X	X	X	X	11	
15.....									X		X		X		X				X	X	X		7	
Number missed	0	0	0	0	0	0	0	0	1	1	2	1	4	2	2	1	2	0	2	4	2	1	25	0
Measurements:																								
16.....											X					X		X	X		X	X	6	
17.....				X	X			X		X	X	X		X	X		X		X		X	X	12	
18.....									X	X	X			X		X		X	X			X	8	
19.....					X	X	X			X	X	X	X		X	X	X	X		X		X	13	
20.....						X			X									X			X	X	5	
Number missed	0	0	0	1	2	2	1	1	2	3	4	2	1	2	2	3	2	4	3	1	3	5	44	0
Percentage:																								
21.....					X								X					X			X	X	5	
22.....										X		X						X	X	X		X	6	
23.....			X						X			X		X		X	X	X	X	X	X	X	11	
24.....		X								X	X					X	X		X	X	X	X	9	
25.....							X		X				X	X		X		X	X	X	X	X	10	
Number missed	0	1	1	0	1	0	1	0	2	2	1	2	2	2	0	3	2	4	4	4	4	5	41	0
Total number missed	0	1	1	3	4	4	5	7	8	9	9	9	10	11	11	11	11	12	12	14	14	21	187	0

*The numbers in the first column refer to the various
horizontal row refer to individual teachers who too

Table 2
ich Individual Teachers Made Errors*

21	22	Total	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	Total
			Teacher, 1973																						
X	X	7									X		X		X	X		X				X			6
	X	8													X		X								2
X	X	8								X			X				X					X			4
	X	5										X						X						X	3
X	X	13														X	X					X	X		4
3	5	41	0	0	0	0	0	0	0	1	1	1	2	0	2	2	3	2	0	0	0	3	1	1	19
X	X	8															X		X				X		3
X	X	15																			X		X	X	3
	X	4													X			X		X	X		X	X	6
	X	8					X							X	X			X		X	X		X	X	8
	X	1																			X		X		2
2	5	36	0	0	0	0	1	0	0	0	0	0	0	1	2	0	1	2	1	2	4	0	5	3	22
		4									X					X			X	X		X		X	6
		3																		X				X	2
		0																							0
X	X	11										X	X	X		X		X	X	X	X	X	X	X	11
X		7																	X		X			X	3
2	1	25	0	0	0	0	0	0	0	0	1	1	1	1	0	2	0	1	3	3	2	2	1	4	22
X	X	6										X													1
X	X	12															X					X			2
	X	8						X											X			X			3
	X	13			X	X	X	X	X	X		X	X	X	X		X	X	X	X	X	X	X	X	18
X	X	5														X									1
3	5	44	0	0	1	1	1	2	1	1	0	2	1	1	1	1	2	1	2	1	1	3	1	1	25
	X	5											X	X							X		X	X	5
	X	6							X	X						X			X	X		X		X	7
X	X	11							X		X			X	X			X	X	X	X	X	X	X	11
X	X	9													X	X	X			X	X	X	X		7
X	X	10									X	X	X	X		X	X	X	X	X	X	X	X	X	13
	5	41	0	0	0	0	0	0	2	1	2	1	2	3	2	3	2	2	3	4	4	4	4	4	43
	21	187	0	0	1	1	2	2	3	3	4	5	6	6	7	8	8	8	9	10	11	12	12	13	131

refer to the various examples listed in Table 1, those in the top
idual teachers who took the diagnostic-survey test.

Table 3

Results of t- tests for the Entire Top, Bottom, and Middle Groups.

(t* = significant difference .01 level)

Number Missed		1930	1973	
entire	mean	8.50	5.95	
	s.d	4.95	4.05	t = 1.83*
	n	22	22	df = 42
top 27%	mean	2.17	1.00	
	s.d.	1.57	.82	t = 1.48
	n	6	6	df = 10
bottom 27%	mean	14.00	11.17	
	s.d.	3.32	1.34	t = 1.77
	n	6	6	df = 10
middle	mean	8.30	5.20	
	s.d.	2.24	2.04	t = 3.07*
	n	10	10	df = 18

Table 4

Results of t-tests on Subsections of the Exam

(t* = significant difference .01 level)

Number Missed

	1930	1973	
Whole Numbers			
mean	1.86	.86	
s.d.	1.29	1.01	t = 2.80*
n	22	22	df = 42
Fractions			
mean	1.64	1.00	
s.d.	1.23	1.41	t = 1.57
n	22	22	df = 42
Decimals			
mean	1.14	1.00	
s.d.	1.22	1.17	t = .380
n	22	22	df = 42
Measurement			
mean	2.00	1.14	
s.d.	1.31	.69	t = 2.66*
n	22	22	df = 42
Percentage			
mean	1.86	1.95	
s.d.	1.56	1.49	t = -.194
n	22	22	df = 42

Table 5

Results of t-tests on Subsections
for Top, Bottom, and Middle Groups

Number Missed

		mean s.d. n	1930	1973	
	I. Whole Numbers				
	II. Fractions				
	III. Decimals				
	IV. Measurement				
	V. Percent				
Top 27%	I.		.50 .50 6	0 0 6	t = 2.36* df = 10
	II.		.33 .47 6	.17 .37 6	t = .598 df = 10
	III.		0 0 6	0 0 6	t = 0 df = 10
	IV.		.83 .90 6	.83 .69 6	t = 0 df = 10
	V.		.50 .50 6	0 0 6	t = 2.236* df = 10
Bottom 27%	I.		3.00 1.00 6	1.17 1.34 6	t = 2.45* df = 10
	II.		2.33 1.25 6	2.50 1.71 6	t = -.179 df = 10
	III.		1.83 1.21 6	2.50 .96 6	t = -.970 df = 10

continued on next page

Table 5 (continued)

		1930	1973	
	IV.	3.00	1.50	
		1.29	.76	t = 2.24*
		6	6	df = 10
	V.	3.83	3.83	
		.90	.37	t = 0
		6	6	df = 10
Middle	I.	2.00	1.40	
		1.00	.92	t = 1.32
		10	10	df = 18
	II.	2.00	.60	
		.89	.80	t = 3.51*
		10	10	df = 18
	III.	1.40	.70	
		1.11	.64	t = 1.64
		10	10	df = 18
	IV.	2.10	1.10	
		.94	.54	t = 2.77*
		10	10	df = 18
	V.	1.50	2.00	
		.92	.63	t = -.1.35
		10	10	df = 18